

**Intuitive Compounding: Framing, Temporal Perspective, and Expertise**

Eric M. Eisenstein,\* Cornell University, eme23@cornell.edu

Stephen J. Hoch, University of Pennsylvania, hochs@wharton.upenn.edu

December 2007

**WORKING PAPER. Do not cite or circulate without permission from the authors.**

Eric M. Eisenstein is an Assistant Professor at the Johnson School, Cornell University, and Stephen J. Hoch is the Patty and Jay H. Baker Professor of Marketing and Director of the Jay H. Baker Retailing Initiative at the Wharton School, University of Pennsylvania. This research was funded by Cornell University and by the Wharton School.

Correspondence concerning this article should be addressed to  
Eric M. Eisenstein  
Johnson School, Cornell University  
326 Sage Hall, Ithaca, New York 14850-6201  
Phone: 607-255-8627  
Fax: 607-254-4590  
Email: eric.eisenstein@johnson.cornell.edu

## ABSTRACT

A proper understanding of compound interest is essential for good financial planning. In three experiments, we demonstrate that most people estimate compound interest by anchoring on simple interest and insufficiently adjust upward. This results in large prediction errors, particularly when the timeframe is long or when the interest rate is high. Expert individuals use a different strategy, often referred to as the Rule of 72, that is much more accurate. Regardless of strategy, accuracy is asymmetric. Prospective predictions are easier than retrospective estimates. Finally, we demonstrate that it is possible to substantially improve people's accuracy by using a short training procedure, which has little cost of use.

Draft

“Compounding is the most powerful force in the universe.” – Albert Einstein  
(attributed)

We all have seen the full-page advertisements in the Wall Street Journal from mutual funds and other financial services firms touting the benefits of long term investments, either in a specific fund that has performed abnormally well over a particular time horizon or an investment in a broad-based index. Typically a graph such as Figure 1a would portray the amazing growth of a \$10,000 hypothetical investment at 10%/annum (the typical long-term rate of return assumed for equities since the depression) over a 45 year time horizon.

-----  
Insert Figures 1a and 1b about here  
-----

The magic of compounding indicates that such an investment would return a whopping 7,200% or an average of 160%/year. A closer look at the graph, however, indicates that almost 4,500% of the total 7,200% return happens in the last 10 years of the investment, not exactly the fact on which the advertisement wants the reader to focus attention.

We argue that the effectiveness of these types of ads is due to the fact that they partially educate consumers about selective outcomes due to compounding without really giving consumers any education about the mechanics of compounding. Figure 1b shows a more complete view of compounding and illustrates that astronomical growth in an investment depends not only on the length of the investment but also the rate of return. Although the authors understand the math behind the chart, it is difficult to imagine that we and most readers would not systematically underestimate the total investment values, especially as time horizons get longer and rates of return get larger.

Many important financial decisions hinge on a proper understanding of compound interest.

For example, decisions on how much and when to invest in a 401(k) plan, or whether to refinance a loan and how to trade off various attributes of a loan instrument (rate, points up front, duration, etc.), cannot be made without a proper understanding of compounding. Furthermore, some judgments that enter into assessments of well-being and happiness, such as whether prices or wages are fair (e.g., Bolton, Warlop, and Alba 2003), also depend on a proper understanding of compounding. When consumers approach such decisions, however, their understanding of this critical concept is frequently quite poor. This problem is exacerbated because the geometric nature of compounding is counterintuitive even to those who are intellectually familiar with the underlying theory. Even sophisticated consumers who understand the theory do not whip out calculators for every possible financial decision, and they certainly do not immediately turn to a computer when making judgments about inflation or price or wage fairness. Furthermore, to accurately choose an option in many of the financial domains outlined above would require greater spreadsheet modeling expertise than is possessed by the vast majority of consumers. All of these observations lead to the conclusion that there will be a substantial intuitive component to consumers' decision-making processes when problems are fundamentally based around compounding processes.

Consider, for example, the following two retirement savings examples:

**Scenario A:** Barney, 20 years old, puts \$1,000 into an IRA that returns on average 12 percent a year

**Scenario B:** Fred spends his money in his 20's, and puts his \$1,000 into an IRA earning 12% at the age of 30

How much worse off is Fred in retirement? The answer is that for every dollar not invested at age 20, Fred is \$45 poorer at retirement, or only 38% as wealthy as he could have been if he had followed Barney's lead. Now consider the following examples:

**Scenario C:** Betty, 20 years old, puts \$1,000 into an IRA that returns 12% per year.

**Scenario D:** Wilma, 20 years old, puts \$1,000 into an IRA that returns 10% per year. How much worse off is Scenario D? For every dollar invested in the 10% fund, Wilma is \$91 poorer at retirement, or only 44% as wealthy as she could have been if she had invested in the same investment as Betty. If people underestimate the effects of time or interest rate (or worse, both), they underestimate the penalty associated with delaying investment or of investing at lower interest rates, leading to poorer decisions. Worse, if people perceive the differences as substantially smaller than they truly are, they are unlikely to turn to a calculator or computer to work out the answer.

A limited number of studies (e.g., Jones 1979; Keren 1983; Wagenaar and Sagaria 1975; Wagenaar and Timmers 1978, 1979) have investigated people's ability to intuitively predict the outcome of an exponential growth process. These studies are consistent in finding that people have a difficult time with such tasks, and that people generally underestimate growth. However, much of this research was conducted in unfamiliar domains, such as the growth of duckweed on a pond, rather than in the more familiar framework of financial investment returns. The choice of an unfamiliar domain is likely to reduce the ability of participants to transfer any expertise that they might possess – an effect that has been demonstrated repeatedly in other tasks such as the Wason 4-card selection task (Hoch and Tschirigi 1983). Furthermore, previous research has not examined differences in strategy across subjects, and we therefore know little about what psychological processes are used by subjects to generate their estimates.

In this research, we examine people's perceptions of exponential growth in its most familiar domain, compound interest. We specifically look for expertise effects and for heterogeneity in strategy use. In particular, we expect heterogeneity to arise due to experience. For instance, many people work with compound interest all the time, either as part of their job, or because they invest on their own, and these people might develop substantial expertise in estimating the

results of compounding. In addition, some people may rely upon remembered heuristics such as “7% doubles in 10 years and 10% doubles in 7 years”, or the more generalizable “Rule of 72,” which should improve accuracy.

### *The Mathematics of Compound Interest*

Given an interest rate,  $i$ , and a time horizon,  $t$ , the formula governing interest compounded at discrete intervals is

$$FV = PV(1 + i)^t, \quad (1)$$

where  $FV$  and  $PV$  are, respectively, the future value and present value of the investment,  $i$  is the interest rate per period, and  $t$  is the number of periods or term.

When estimating the results of a compounding process in one’s head, it is nearly impossible to use Equation 1 directly, as the amount of multiplication required is simply too great. We hypothesize that most people ignore compounding, and instead anchor on simple interest,

$$FV = PV(1 + i t), \quad (2)$$

here  $FV$ ,  $PV$ ,  $i$ , and  $t$  have the same meanings as in Equation 1. Anchoring on simple interest yields acceptable accuracy for very short time periods and for “low” interest rates. However, massive underestimation results when the time period is long or when interest rates are higher, because compounding is an exponential process and simple interest is linear. Most of the major financial decisions that people make during their lives involve either long time periods (e.g., retirement saving, mortgages) or high interest rates (e.g., taking on credit card debt), which makes anchoring on simple interest a pernicious bias.

Anticipating our results, a small number of people achieve much higher levels of accuracy by using an approximation to Equation 1, which is called the “Rule of 72.” The core of the Rule

of 72 approximation is the realization that an initial value will double at a constant time interval if the interest rate remains constant. Specifically, if  $y$  is the number of years that it takes for money invested (or loaned) at an interest rate of  $i$  to double in value, then  $y = 72/i$ .<sup>1</sup> Once the doubling time is known, the number of doublings,  $d$ , can be derived by dividing the term by the doubling time,  $d = t/y$ , and therefore  $FV = PV * 2^d$ . For example, if \$100 is invested for 24 years at 9% interest,  $y = 72/9 = 8$  years,  $d = 24/8 = 3$  doublings, and  $FV = \$100 * 2^3 = \$800$ . The exact answer is \$791. For rates of return less than 14% and an investment horizon less than 25 years, the Rule of 72 produces an answer within 10% of the correct total return.

We conducted three experiments examining the intuitive compounding task with current undergraduate students, MBA students, and ordinary people from an Internet panel. The first study showed that over 90% of people anchored on simple interest, insufficiently adjusted, and had an average prediction error ranging from 10-80% depending on interest rate and term. A small group of subjects indicated reliance on the Rule of 72, which more than halved their prediction error. Study 2 compared current MBAs to ordinary consumers and had respondents make prospective (e.g., a 9% interest rate, 16 years in the future) versus retrospective (e.g., a 13% interest rate, 8 years in the past) predictions. Again, prediction errors of Rule of 72 respondents were almost 50% smaller compared to people relying on adjustments away from simple interest. MBAs were no more accurate than ordinary people and prospective predictions were one third more accurate than retrospective ones. Finally, in Experiment 3, we examined the

---

<sup>1</sup> Why 72/? To find the doubling time, set  $FV = 2$  and  $PV = 1$ , such that  $2 = 1 * (1+i)^t$ , and solve for  $t$ . Taking logs,  $t = \ln(2) / \ln(1 + i)$ . Using the Taylor series expansion of  $\ln(1+i)$ , we find that  $\ln(1+i) = i - \frac{i^2}{2} + \frac{i^3}{3} - \frac{i^4}{4} + \dots$ , thus,  $\ln(1 + i) \approx i$  for small values of  $i$ . Since  $\ln(2) = .69$ , the exact mathematical Rule is “the Rule of 69,” which is indeed appropriate for very small values of  $i$  (as typically encountered in continuous compounding). However, ordinarily encountered interest rates are “not small,” and error is minimized in the range of typical annual interest rates by increasing the numerator slightly (using 72 inflates the answer, which is similar to the effect of using the second term of the Taylor expansion). For very high interest rates one could adjust the numerator up further (e.g., at 20%, money doubles in 3.8 years, so it would be better to use  $76/.20 = 3.8$  years, rather than  $72/.20 = 3.6$  years).

remedial impact of a simple 5 minute procedure to train people to use the Rule of 72. Training reduced prediction errors by about 45% while increasing response times from 40 to 45 seconds (12%). Both undergraduates and ordinary consumers benefited from the training, though the undergraduate sample benefited more. Finally, we observed that the training successfully transferred to a related but different problem involving compounding.

### **General Procedure**

In these experiments, we explored the heuristics and biases associated with estimates of compound interest. We examined differences in processing arising from negative vs. positive framing (e.g., debt vs. investments), differences between retrospective and prospective compounding estimation (i.e., what was the cost of a TV in 1970? vs. what will be the cost of a TV in 2030?), and the effects of expertise and training on accuracy.

All experiments followed a similar procedure. Data were collected over the Internet. Each study began with an introduction that explained the concept of compound interest and its importance, indicated that subjects should be in a quiet, disturbance-free area, and that calculators, paper and pencil, and other aids were not permitted. Compound interest was explained in the following way:

Compound interest means that when interest is earned, it is left in the account. In future years, interest accumulates on the full amount that is in the account, so you “earn interest on the interest” as well as on the original principal amount. This is what we mean by compound interest, or “compounding the interest.”

After this introduction, a cover story was presented that set up the compounding problems.

Subjects responded to multiple compounding problems, which were presented serially. On each trial of the study, subjects were presented with both text and a table containing the values of the three attributes of the stimulus (i.e., interest rate, amount of money, and term). A typical stimulus (in abstract form; subjects saw an actual amount, interest rate, and term) was:

You currently have a balance of  $\$X$  in your account. You leave this money in your savings account for  $t$  years at a constant interest rate of  $t\%$  per year. Interest is compounded annually. All interest is reinvested into the account. During this time, no additional money other than interest is ever put into or removed from the account.

Below the verbal description, the attributes were displayed in a table. Subjects were told to type in their best estimate of the result of the compounding process. After responding, the next stimulus was displayed and the process repeated until subjects had responded to all of the problems. There was a 1 second pause between stimuli, during which the screen went blank except for the words “Loading next question...”. Subjects were given two practice problems before responding to the study questions.

These studies used multiple vectors of initial values that were assigned to each combination of interest rate and term, and multiple orders of presentation. Value-vectors were composed by using a subset of values from \$1,000 to \$9,000 in \$1,000 increments. Depending on the number of stimuli, each value could appear more than once, but all values appeared an equal number of times. Value-vectors were constructed so that each vector of values was orthogonal to the vectors of interest rates and durations, and each value-vector was also orthogonal to the other value-vectors. There were always at least two orders of presentation, typically a random order and its reverse.

Response latency was captured on every trial, and subjects were unaware that latency would be measured. After completing the main study, subjects completed a series of additional questions, which included a scale measuring financial knowledge, some demographic information, and an open-ended question that asked subjects how they produced their estimates.

#### *Internet research*

All data was collected over the Internet. Internet studies have numerous benefits, but they sacrifice some degree of control, and thus it becomes important to clean the data before analysis

(see Birnbaum 2004 for a review of the issues arising from Internet-based data collection). Accordingly, subjects were removed from the analysis according to the following specific criteria. First, the vast majority of the eliminations stem from subjects who improperly used calculators. Subjects were classified as having used a calculator if they had any one answer exactly correct including decimal values, if they had more than one answer exactly correct when rounded to the nearest integer, or if the majority of their answers were within round-off error of being exact. Second, subjects were removed from the analysis for taking an impossibly short time to complete the study. In every experiment, there was a bimodal (frequently nonoverlapping) distribution of response latencies. Generally, the gap occurred at approximately the shortest 5% of times. Across experiments, those eliminated for being too fast spent a median time per problem of 1-3 seconds, compared to typical subjects who had median times in the 20-40 second range. Finally, subjects were removed from the analysis for failing to follow directions. Examples include subjects whose open-ended responses stated that they had randomly guessed the answers and subjects who responded with impossible values (such as estimates that were less than the principle amount). Large fractions of excluded subjects would have been excluded based on multiple criteria. Across all studies, approximately 15% of participants were disqualified, which is consistent with typical rates observed in Internet-based research (Birnbaum 2004).

### **Experiment 1**

The purpose of Experiment 1 was to investigate the general way that people estimate compound interest in their head. We predicted that people would anchor on simple interest, resulting in underestimation of the result of compounding, particularly at high interest rates and over long time spans. However, most people have been exposed to frequent admonitions about

the dangers of credit card debt. We compared subjects' estimates of compounding in a debt (credit card) environment to those in a savings environment, with the expectation that estimates would be greater in the credit card domain.

### *Method*

*Subjects.* Subjects were recruited from an introductory undergraduate business course and participated for partial course credit. The data validation criteria outlined in the General Procedure were applied, resulting in a total of 96 subjects, and an elimination rate of 17%.

*Procedure.* The General Procedure was followed, with the following specifics. The design was within-subject, with the exception of order. Each subject provided 24 estimates of the result of a financial compounding process. Stimuli consisted of 24 problems consisting of the full factorial of 3 (interest rate: 4%, 9%, 14%) x 2 (compounded amount: \$1,000 or \$5,000) x 2 (term: 12 or 24 years) x 2 (domain: savings or credit card). The stimuli were presented in a random order and its reverse.

This study compared a savings and a credit card domain. For the credit card problem, subjects were told:

Your credit card company has sent you an offer regarding your credit card: they will permit you to make NO monthly payments of any kind for a number of years. In return, you will not be permitted to charge additional amounts to the card. At the end of the time period, you will pay the credit card company the accumulated balance. You accepted the offer.

The savings problem was similarly phrased, ruling out deposits and withdrawals. Appropriate modifications to the stimulus phrasing described in the General Procedure were made to accommodate the extra domain variable, which was also displayed in the table. On each trial, subjects were told to type their best estimate of what the accumulated balance would be at the end of the specified time period.

### *Results*

*Accuracy.* Analysis of accuracy presented several challenges. First, the actual predictions made by subjects span a very wide range (the minimum correct answer was  $\approx$  \$1,600, the maximum was  $\approx$  \$116,000). Using accuracy measures such as mean absolute deviation (MAD) would emphasize stimuli with greater outcome values, essentially eliminating any effect of small-valued problems on accuracy. Percentage error weights all problems equally, and that is the measure we adopted. The second challenge was that the data contain occasional within-subject outliers. There are a number of reasons that an response might be an outlier (usually by an order of magnitude), with typographical errors being the most common. Within-subject responses were eliminated from the analysis if the absolute percentage error for a given response was greater than 3 standard deviations above an estimate of that subject's mean absolute percentage error.<sup>2</sup> The resulting absolute percentage error data remain right-skewed, and therefore median absolute percentage error (*MedAPE*) was used to characterize central tendency.

Accuracy in prediction showed considerable variation across the sample, with the mean *MedAPE* = .30 across subjects, and a range from .03 to .72. In examining the distribution of accuracy, it was clear that a small group of subjects were considerably more accurate than most of the others. Further analysis of these subjects' open-ended responses to the question, "please explain, in your own words, how you attempted to answer the questions you were presented with. Be as detailed as possible," indicated that they used the Rule of 72 to produce their estimates. An RA, who was unfamiliar with the hypotheses of the research, classified all subjects' open-ended responses into three strategy classes: simple interest, Rule of 72, and "other." Classifications depended only on the response to the open-ended question; the RA had

---

<sup>2</sup> Outlier data points inflate estimates of each subject's mean and standard deviation. Therefore, robust measures of central tendency and dispersion were used. All reported results are robust to reasonable choices for outlier assessment methodology. A total of 2.7% of observations were excluded, and the average was .64 observations per subject (see Eisenstein and Hutchinson Forthcoming; and West 1996, for a similar procedure involving latency times).

no additional information available to her. Respondents were classified as having used the Rule of 72 based on whether the subject's response explicitly mentioned either the Rule of 72 or an equivalent procedure (e.g., 10% interest doubles in 7 years). Subjects were classified as using simple interest if they stated that they used simple interest or an equivalent procedure (e.g., the formula for simple interest). A total of 6% of subjects (6/96) were classified as having used the Rule of 72. Anticipating the results of this and future experiments, we found no differences between subjects classified as "other" and those classified as "simple interest," and therefore, these two categories were combined.

A repeated measures ANOVA was used to compare accuracy across strategies. In addition, we analyzed the within-subject effects of interest rate and term, which are the primitives that drive the true answer to compounding problems. In the analysis of accuracy, we controlled for fatigue, time spent thinking, and financial expertise, hypothesizing that fatigue would reduce accuracy, while additional time spent thinking and expertise would increase it. We controlled for fatigue effects by adding linear and quadratic terms representing the ordinal position in which the stimulus was presented to the model. We measured financial knowledge using each subject's Z-score on a six-item scale of financial knowledge ( $\alpha = .83$ ), which included questions such as "I am experienced at using financial information to make decisions," "I am uninformed about financial matters," and "I invest money myself." We also added each subject's median time taken per problem to capture the amount of time spent on task. Thus, we added a total of four effects into the model, linear and quadratic effects of ordinal position to control for fatigue, a measure of financial knowledge, and each subject's median time taken per problem. There are no qualitative differences that result from including these control variables, but they do allow us to explore additional constructs of interest. To reduce the skew of the outcome variable, we used

the logarithm of absolute percentage error as the dependent variable. For ease of interpretation, we estimated the model using a log-log specification, and least square mean estimates from the model have been reported as median absolute percentage error (denoted *lsm* throughout).<sup>3</sup>

-----  
Insert Figure 2 about here  
-----

As expected, subjects employing the Rule of 72 were more accurate ( $n = 6$ ;  $lsm = .10$ ) than subjects employing simple interest strategies ( $n = 90$ ;  $lsm = .26$ ),  $F(1, 92) = 10.3, p = .002$ . There was no effect of domain on accuracy (main effect and interaction  $F$ 's  $< 1$ ). All subjects made greater errors as interest rates increased,  $F(1, 2134) = 617, p < .0001$ , and as term increased,  $F(1, 2134) = 326, p < .0001$ . Figure 2 displays the least square mean estimates of accuracy for all combinations of term and interest rate, and reveals that subjects using the Rule of 72 were more accurate at 24 years than subjects using simple interest were at 12 years. The effects of term and interest rate on accuracy were approximately equal, which is shown by the fact that the elasticity of error with respect to interest rate was 1.18, and the elasticity with respect to term was 1.27. The elasticities reveal that a 10% increase in rate would result in a 18% increase in absolute percentage error. Similarly, a 10% increase in term would result in a 27% increase in error. These elasticities did not vary by strategy. Coefficients for financial knowledge and time were in the expected directions (i.e., more knowledge and more time reduce error). Subjects who scored higher on the financial knowledge scale were more accurate,  $F(1, 92) = 14.2, p = .0003$ , regardless of which strategy they used, with an effect size amounting to a 25% reduction in error per standard deviation increase in their score. The effect of time on error

---

<sup>3</sup> There are no qualitative differences in any reported results between semi-log and log-log specifications of the model. The antilog of predicted values from a regression in which the dependent variable is in log space produces an asymptotically unbiased estimate of the median of the  $Y$  distribution (Goldberger 1968).

was nonsignificant, and no effects of order were found for any outcome variable of interest.

*Time.* Subjects who used the Rule of 72 to produce their estimates spent only directionally more time to produce their estimates than subjects employing other, less accurate, techniques. Log time per problem was regressed onto the between subject variables and the non-temporal control variables used in the analysis of accuracy (i.e., fatigue and financial knowledge). Subjects using the Rule of 72 ( $lsm = 26.0$  s) spent approximately four seconds more time than subjects employing a simple interest strategy ( $lsm = 21.1$  s),  $F(1, 93) = 1.12, p = .29$ , demonstrating that there was little cost associated with using the more accurate strategy.<sup>4</sup>

*Process.* We classified subjects as having used either the Rule of 72 or simple interest based on self-reports. It is not always the case that subjects can explain in words the process used to generate estimates (Nisbett and Wilson 1977). We can better characterize the strategies employed by subjects by fitting an “as if” model. Wagenaar and Timmers (1975) proposed a model of how people estimate exponential series,  $FV = \alpha PV (1 + r)^\beta$ , where  $\beta$  was interpreted to represent a constant degree of underestimation of the exponential term, and  $\alpha$  represented a multiplier that is used to increase the size of estimates. This model is more conveniently expressed as

$$FV / PV = \alpha (1 + r)^{\beta t}, \quad (3)$$

which we use throughout. We fit this model at the individual level to compare the coefficients across classification strategies, and to gain insight into subjects’ thought processes.

Equation 3 is sufficiently flexible that it will provide an excellent fit to data generated by a pure simple interest process for a given dataset (i.e., data generated using Equation 2). If data were generated from Equation 2, then the least squares estimates of the parameters of Equation 3

---

<sup>4</sup> Latency data is right skewed. Following West (1996) and Eisenstein and Hutchinson (Forthcoming), times that were greater than 3 SD above each subject’s mean (measured using robust methods) were excluded from analysis (see also Fazio 1990). The percentage of data satisfying this condition was less than 5% across all experiments.

would be  $\alpha = .35$ ,  $\beta = .36$ , with an  $R^2 = .98$ . By contrast, if subjects responded with perfectly computed Rule of 72 answers, then the estimated parameters will be very close to  $\alpha = 0$ ,  $\beta = 1.00$ , which are the parameters associated with a pure compounding model.

-----  
Insert Figure 3 about here  
-----

We estimated Equation 3 for each subject, and used a repeated measures ANOVA to compare the coefficients for the Rule of 72 and the simple interest groups. The resulting least square means for each coefficient are plotted in Figure 3, along with the structural parameters that result from fitting Equation 3 to data generated using simple interest (i.e., Equation 2) and the coefficients expected from a pure compounding process. As predicted, the pattern of coefficients differed by strategy,  $F(1, 95) = 17.8$ ,  $p < .0001$ . Figure 3 reveals that subjects' coefficients correspond closely to the structural parameters for their respective strategies, validating the classification procedure.

### *Discussion*

Experiment 1 confirms that there is substantial heterogeneity in the accuracy of prediction associated with an exponential processes. These accuracy differences are largely driven by a learned strategy, the Rule of 72, that some subjects had previously learned. Subjects using the Rule of 72 were more than twice as accurate as subjects using other strategies, and this accuracy difference was not explained by additional financial knowledge or additional time spent thinking. The correspondence of the structural and individual-level estimated coefficients confirmed that our classification of subjects into strategies based on subjects' open-ended comments was reasonable. Furthermore, the individual-level coefficient estimates revealed that subjects who

did not use the Rule of 72 strongly anchored on simple interest, which resulted in large errors. These errors remained (and were directionally greater) for the stimuli with the greatest answer magnitudes (i.e., those with the greatest interest rate and duration).

We expected that the frequent warnings about the dangers of credit card debt might have influenced subjects to estimate greater values when problems were in a credit card rather than savings domain. This hypothesis was not supported, which may have occurred for two reasons. First, subjects were all undergraduates studying business, and they might have explicitly learned that domain is irrelevant, and therefore not paid attention to it. Second, the within-subject nature of the design makes such an effect more difficult to find. Overall, however, Experiment 1 demonstrated that some people can be quite accurate in predicting the results of an exponential process, and that these people are very likely to have been using a wholly different strategy for generating estimates than their less accurate peers.

## **Experiment 2**

In Experiment 2, we investigated the effects of expertise and temporal frame on subject's estimates of compound interest. Expertise was captured by using a sample of financially sophisticated MBA students, and contrasting them with a diverse sample of people from different backgrounds. Temporal frame refers to whether the goal is to estimate present or future value. For example, consumers may need to estimate the effects of inflation on the price of a good or on wages (Bolton et al. 2003). Such problems can have either a *prospective* or a *retrospective* temporal frame. Assume that the time horizon is 20 years and that the inflation rate is 3%. In the prospective frame the question would be: "if a good costs \$X today, how much will it cost in 20 years?" In the retrospective frame the question would be: "if a good costs \$X today, how much did it cost 20 years ago?" Mathematically, these alternatives simply indicate whether \$X

represents  $PV$  or  $FV$  in Equation 1.

We anticipated that subjects would be less accurate in the retrospective frame than in the prospective frame. This prediction is based on the fact that the operations required to solve for  $PV$  in Equations 1 and 2 are not the same as the operations required to solve for  $FV$ . In the simple interest case (Equation 2), solving for  $FV$  involves a series of multiplication and addition steps that are quite familiar (i.e., compute one year of interest,  $PV * i$ , multiply this by  $t$ , and add back the principle,  $PV$ ). By contrast, solving for  $PV$  requires more complex, less intuitive, steps to be carried out, and requires a difficult division problem as well (i.e., first multiply  $i * t$ , add one, and then divide this result, which may be a decimal, into  $FV$ ). In the case of the Rule of 72, things are somewhat simpler. Because the interest rate and duration together determine the number of doublings, solving for  $PV$  becomes a matter of successively halving the  $FV$  (or of dividing  $FV$  by  $2^d$ ). We believe that this process of successive halving will be more difficult than successive doublings, but it is likely to be easier than the simple interest process.

#### *Method*

*Subjects.* Subjects were recruited from an Internet panel of “ordinary people” and from the MBA class at a Northeastern university. All participated for a payment of \$7. The data validation criteria outlined in the General Procedure were applied, resulting in a total of 132 participants, and an elimination rate of 14%. Of the total, 100 subjects came from the Internet panel and 32 were MBA students. The mean age of the Internet sample was 40 and the mean of the MBA sample was 31. Only 46% of the Internet sample had graduated from college, and 80% of those had earned a bachelors or associate degree. By contrast, 100% of the MBA sample held a bachelors degree (and all were pursuing a masters).

*Procedure.* The general procedure was followed, with the following specifics. The design

was a mixed within-between factorial. Each subject provided 16 estimates of the result of a compounding process in the financial domain, the stimuli for which were composed from the full factorial of 4 (interest rate: 5%, 9%, 13%, 18%) x 4 (duration: 8, 16, 24, 32 years). Subjects were randomly assigned to one cell of a factorial design consisting of 2 (temporal frame: prospective or retrospective) x 3 (value vector: 1, 2, or 3) x 2 (order: 1, 2). On each trial in the prospective condition, subjects were given the present value of an investment and estimated its future value at the end of the investment duration. On each trial in the retrospective condition, subjects were given the future value of the investment (the value today), and estimated the present value (i.e., the initial investment amount that would generate the presented value at the displayed interest rate and duration). There were three vectors of initial values that were assigned to each of the 16 stimuli. Each vector consisted of eight values ranging from \$2,000 to \$9,000 in \$1,000 increments, with each of the eight dollar amounts appearing twice. The values were assigned to the 16 stimuli so that each vector of values was orthogonal to the vectors of interest rates, durations, and to the other value vectors. Stimuli were presented in a random order and its reverse. Appropriate wording changes were made to the example stimulus in the General Procedure section to account for temporal frame.

### *Results*

*Accuracy.* We used the same modeling and data-cleaning procedures as in Experiment 1.<sup>5</sup> Following the same procedure that was used in Experiment 1, an RA classified all subjects' open-ended responses into strategy classes, and the "other" and "simple interest" groups were combined in the analysis due to nonsignificant differences between them. Ultimately, 23 subjects used the Rule of 72, and 109 subjects were classified as having used simple interest.

As in Experiment 1, we computed a repeated measures ANOVA to compare accuracy across

---

<sup>5</sup> Excluded data was 6% of all observations, and the average was .96 observations per subject.

strategy and temporal frame. Preliminary analysis revealed that three subjects in the simple interest group had errors so far above their peers that they materially distorted the estimates of the means and statistical tests, and these subjects were excluded from further analyses.<sup>6</sup> As predicted, subjects employing the Rule of 72 were more accurate ( $n = 23$ ;  $lsm = .28$ ) than subjects employing simple interest strategies ( $n = 109$ ;  $lsm = .51$ ),  $F(1, 119) = 15.1, p = .0002$ , and the prospective frame ( $lsm = .31$ ) was more accurate than the retrospective frame ( $lsm = .47$ ),  $F(1, 119) = 8.32, p = .005$ . The strategy x temporal frame interaction was not significant. Although MBA students performed better than ordinary people when both groups used a simple interest strategy, the reverse was true when the strategy used was the Rule of 72, as shown by the strategy x sample interaction,  $F(1, 119) = 5.21, p = .02$ .

As in Experiment 1, the observed differences in accuracy across strategies held for stimuli with the greatest outcome magnitudes, which are of the greatest importance to financial well-being. All subjects made greater errors as interest rates increased,  $F(1, 1662) = 549, p < .0001$ , and as term increased,  $F(1, 1662) = 686, p < .0001$ . The effects of term and interest rate on accuracy were equal. The elasticity of error with respect to interest rate was .88, and the elasticity with respect to term was .91. The elasticities reveal that a 10% increase in rate would result in an 8.8% increase in absolute percentage error. Similarly, a 10% increase in term would result in a 9.1% increase in error. These elasticities mask large effects due to interest rate and term. In this experiment, term varied from 8 to 32 years. Moving from 8 to 16 years represents a 100% increase in term, which would result in a 91% increase in absolute error. Similarly, interest rates varied from 5% to 18%, with correspondingly large predicted increases in error. As expected, coefficients for financial knowledge and time were in the expected directions (i.e., more knowledge and more time reduced error). Subjects who scored higher on the financial

---

<sup>6</sup> The average of these subjects' *median* absolute percentage error was over 600%.

knowledge scale were more accurate,  $F(1, 119) = 6.23, p = .01$ , regardless of strategy, with an effect size amounting to a 13% reduction in error per standard deviation increase on the scale of financial knowledge. Those who took greater time on average for each problem were also more accurate,  $F(1, 119) = 4.91, p = .03$ . There were no effects of order or value-vector on any variable of interest.

*Time.* Log time per problem was regressed onto the between subjects variables and the non-temporal control variables used in the analysis of accuracy. As in Experiment 1, subjects using the Rule of 72 ( $lsm = 21.4$  s) did not take significantly more time than subjects employing a simple interest strategy ( $lsm = 16.1$  s),  $F(1, 127) = 2.40, p = .12$ . Further analysis revealed that there were no differences in time taken due to highest degree attained or major in school.

*Process.* As in Experiment 1, we classified subjects as having used either the Rule of 72 or simple interest based on their self-reports. To delve more deeply into the process used by subjects to produce their responses, we estimated the same model used in Experiment 1 (i.e., Equation 3) for each subject. As noted in Experiment 1, this model is sufficiently flexible that it will provide an excellent fit to data produced by a simple interest model for these data. If data were generated from the simple interest process defined by Equation 2, then the structural least squares estimates of the parameters of Equation 3 would be  $\alpha = .54, \beta = .27 (R^2 = .97)$ . As in Experiment 1, if subjects responded with perfectly computed Rule of 72 answers, then the estimated parameters would be very close to  $\alpha = 0, \beta = 1.00$ , the parameters associated with a pure compounding model.

We used a repeated measures ANOVA to compare the estimated coefficients for the Rule of 72 vs. the simple interest group. The resulting least square means for each coefficient are plotted in Figure 4, along with the structural parameters that result from fitting Equation 3 to data from a

simple interest and a Rule of 72 generating process. The patterns of estimated coefficients differed by strategy,  $F(1, 126) = 22.0, p < .0001$ . No difference in the pattern of coefficients was observed due to temporal frame. Between subjects, the Strategy x Temporal Frame interaction was marginally significant,  $F(1, 126) = 3.43, p = .07$ , reflecting the upward shift in the parameter estimates of the simple-retrospective group. Neither educational level nor major affected the coefficient estimates ( $F$ 's  $< 1$ ). Replicating the results of Experiment 1, Figure 4 reveals that subjects employing “other” strategies are anchoring on simple interest, and that this is particularly true in the prospective temporal frame.

-----  
Insert Figure 4 about here  
-----

As in Experiment 1, subjects classified as using the Rule of 72 appear to be doing an excellent job applying the Rule of 72 regardless of temporal frame. The coefficient pattern of the simple interest/retrospective subjects, which looks like an intercept-shifted simple interest model, arises because subjects in this group thought that initial (i.e., present values) should be lower than a pure simple interest model (but still greater than the true answer). The pattern of coefficients is consistent with a “linear adjustment” model, in which one first computes the answer from simple interest (anchoring on it), then linearly adjusts that answer.

### *Discussion*

Expertise, as measured by level of education or major in school had no direct impact on accuracy. However, those with stronger math and business backgrounds did take less time on average than those with degrees in other fields. As predicted, the retrospective temporal frame was more difficult than the prospective frame, and directionally, the retrospective frame hurt

those using simple interest more than those using the Rule of 72. As in Experiment 1, our classification of self-reported strategies was validated by the correspondence between the average individual-level estimate of the coefficients of Equation 3 and the structural estimates that would have been produced by a perfect application of each strategy. Overall, the results of Experiment 2 replicate and extend those of Experiment 1.

### **Experiment 3**

Experiments 1 and 2 clearly demonstrated that some people across a wide range of backgrounds have been exposed to the Rule of 72, and that they use the Rule of 72 when estimating the results of financial compounding problems. Furthermore, the previous experiments demonstrated that people appear to naturally anchor on simple interest in the absence of using the Rule of 72. Previous experiments also rule out general financial expertise or willingness to spend more time thinking as explanations. In Experiment 3, we investigate whether it is possible to train subjects to effectively use the Rule of 72.

#### *Method*

*Subjects.* Subjects were recruited from an Internet panel of “ordinary people” and from an undergraduate class at a Northeastern university. The data validation criteria outlined in General Procedure were applied, resulting in a total of 197 participants, and an elimination rate of 15%. Of the total, 93 subjects were recruited from the introductory undergraduate business course and participated for partial course credit, and 104 subjects were recruited from the Internet pool of “ordinary people” who participated for a payment of \$9. The mean age of the student subjects was 20, the mean age for ordinary people was 39. Students were all pursuing a bachelors degree; however, 48% of the Internet sample had earned less than a bachelors degree, and 88% had earned less than a masters degree.

*Procedure.* The general procedure was followed, with the following specifics. The design was a mixed within-between factorial. Each subject provided 12 estimates of the result of a compounding process in the financial domain. The 12 stimuli were composed of the full factorial of 3 (interest rate: 9%, 13%, 18%) x 4 (duration: 8, 16, 24, or 32 years). Subjects were randomly assigned to one cell of factorial design consisting of 2 (training: train or control) x 2 (value vector: 1 or 2) x 2 (order: 1 or 2).

Subjects in the training condition were then taught the Rule of 72. First, they were taught how to compute the doubling time, and examples were shown. After reading this procedure, they were presented with 5 practice problems that asked, for example: “You deposit money into a bank account at an interest rate of 12% interest. How many years will it take for the money to double in value?” After entering their response, the correct answer appeared on the screen, for example: “To answer this question, divide 72 by 12. Since  $72/12 = 6$ , it will take 6 years for the money to double.”

After completing the five practice doubling time problems, subjects received instructions on how to compute the number of doublings, and were presented with five two-part practice problems. An example of the first part of each problem was, “If invested money will double every 5 years, how many times will the money double if you invest it for 10 years?” and an example of the second part was, “Given your answer above, if you have \$300 in the bank, how much will the \$300 be worth at the end of the investment (given the number of doublings you worked out)?” After inputting answers to both problems, subjects received feedback underneath their answers: “Answer: The money doubles every 5 years. Thus, it will double  $10/5 = 2$  times,” and “Answer: Above, we found that the money doubles 2 times. Since the money doubles 2 times, we multiply \$300 by 2 that number of times in a row:  $\$300 * 2 * 2 = \$1,200$ .”

Subjects in the control condition performed the exact same mathematical operations as the trained conditions (i.e., dividing a number into 72, doubling it, etc.), but were not told the context of the numerical manipulations. Instead, control condition subjects were told that computing compound interest requires simple math, and that they should “practice math in their head.” Thus, control subjects performed the same number of practice problems, with identical difficulty, as the trained subjects.

### *Results*

*Accuracy.* As in Experiments 1 and 2, we used log absolute percentage error as the accuracy measure, the data were cleansed of within-subject outliers using the same procedures as described in previous experiments,<sup>7</sup> and a repeated measures ANOVA was used to compare accuracy across training condition and sample. As in Experiment 2, preliminary analysis revealed that four subjects had errors so far above their peers that they materially distorted the estimates of the means and statistical tests, and these subjects were excluded from further analyses. Figure 5 displays accuracy as a function of training condition and sample. As hypothesized, trained subjects were more accurate ( $n = 110$ ;  $lsm = .15$ ) than untrained subjects ( $n = 83$ ;  $lsm = .27$ ),  $F(1, 187) = 43.0, p < .0001$ . In addition, subjects in the undergraduate student sample ( $n = 92$ ;  $lsm = .15$ ) were more accurate than the ordinary people ( $n = 101$ ;  $lsm = .27$ ),  $F(1, 187) = 18.9, p < .0001$ , and there was a slightly greater benefit of training for the undergraduates, Training x Sample interaction,  $F(1, 187) = 4.76, p = .03$ .

-----  
Insert Figure 5 about here  
-----

As in previous experiments, the observed differences in accuracy held for stimuli with the

---

<sup>7</sup> Excluded data was 4.0% of all observations, and the average was .48 observations per subject.

with the greatest outcome magnitudes, which are of the greatest importance to financial well-being. Overall, subjects made greater errors as interest rates increased,  $F(1, 2035) = 203, p < .0001$ , and as term increased,  $F(1, 2035) = 137, p < .0001$ . There was an interest rate x training interaction,  $F(1, 2035) = 38.6, p < .0001$ . Specifically, for the control condition, interest rate (elasticity = .93) was as important a driver of error as term (elasticity = .91), but for the trained group, interest rate (elasticity = 2.2) was 2.2 times as important as term (elasticity = 1.0). As in previous experiments, coefficients for financial knowledge and time were in the expected directions (i.e., more knowledge and more time reduced error). Subjects who scored higher on the financial knowledge scale were more accurate,  $F(1, 187) = 19.4, p < .0001$ , regardless of sample or training condition, resulting in a 30% reduction in prediction error for each standard deviation difference on the scale of financial knowledge. Those who took spent more time on each problem were more accurate,  $F(1, 178) = 7.63, p = .006$ . There were no effects of order or value-vector on any variable of interest.

*Time.* Log time per problem was regressed onto the between subjects variables and the non-temporal control variables used in the analysis of accuracy (i.e., question number, to control for fatigue, and a score of financial knowledge). Subjects using the Rule of 72 ( $lsm = 44$  s) did not take significantly more time to produce their estimates than subjects employing a simple interest strategy ( $lsm = 40$  s),  $F(1, 188) = 1.24, p = .27$ . However, ordinary people ( $lsm = 46$  s) spent more time per problem than student subjects ( $lsm = 38$  s),  $F(1, 192) = 5.25, p = .02$ .

*Process.* As in previous experiments, we fit Equation 3 at the individual level. If data had been generated by the simple interest process described by Equation 2, the structural parameter estimates for stimuli used in this experiment would be  $\alpha = .49, \beta = .30$  ( $R^2 = .98$ ). We used a repeated measures ANOVA to compare coefficients across training and temporal frame. The

resulting least square means for each coefficient are plotted in Figure 6, along with the structural parameters that resulted from fitting Equation 3 to data produced by simple interest and Rule of 72 generating processes.

-----  
 Insert Figure 6 about here  
 -----

The pattern of coefficients differed by training condition,  $F(1, 190) = 31.0, p < .0001$ , by sample,  $F(1, 190) = 5.95, p = .02$ , and the training x sample x pattern interaction was significant,  $F(1, 190) = 9.39, p = .003$ . Figure 6 reveals that the source of the significant interaction is that student subjects were better able to learn or implement the training than were ordinary people. In fact, for ordinary people, the differences in the pattern of coefficients between the trained and control groups are only marginally significant,  $F(1, 190) = 3.70, p = .07$ . This difference in pattern is driven by heterogeneity in response to the training, which we discuss further in the Discussion section.

*Effects on Economic Behavior and Choice*

Subjects were asked the following question at the end of the experiment:

Allison and Betty are both saving for retirement. Both of them will retire when they reach the age of 69. Descriptions of their retirement strategy to date appear in the table below. So far, each woman has invested some money in their retirement account. Both Allison and Betty have chosen a retirement investment strategy that will yield a 9% annual return (interest rate) on their investments.

| <b>Description</b>                                      | <b>Allison Betty</b> |         |
|---|----------------------|---------|
| <b>Age</b>  | 21                   | 29      |
| <b>Amount saved for retirement</b>                      | \$6,000              | \$9,000 |
| <b>Expected annual return on retirement investments</b> | 9%                   | 9%      |
| <b>Age at which they will retire</b>                    | 69                   | 69      |

Assume that each woman stops saving for retirement today. So on this day, they have each saved the amount of money listed above under “amount saved for

retirement.” Neither takes any money out of the account until they retire at age 69. Who will have more money available at the time that they retire, Allison or Betty?

-----  
Insert Table 1 about here  
-----

The correct answer is that Allison will have 33% more money than Betty, with total accumulations of \$375,000 and \$283,000, respectively. A logistic regression was computed using training, sample, and financial knowledge to predict subjects’ binary response. As reflected in Table 1, control subjects were more likely to choose the incorrect answer, Betty, than trained subjects,  $\chi^2(1) = 8.22, p = .004$ . Separately, ordinary people also chose the incorrect answer more frequently,  $\chi^2(1) = 3.56, p = .06$ , which is consistent with our finding that ordinary people had a more difficult time learning the Rule of 72. As expected, subjects were more likely to choose the correct answer as financial expertise increased,  $\chi^2(1) = 9.33, p = .002$ .

### *Discussion*

Training significantly reduced error for both ordinary people and undergraduate students; overall, trained subjects were twice as accurate as control subjects. However, the training worked better for students than it did for ordinary people, in part because the increases in accuracy for ordinary people were somewhat muddled by heterogeneity in response to the training. It is clear from Figure 6 that undergraduate students learned or implemented the training procedure better than ordinary people from the Internet sample, because coefficients for the control conditions look as though they lie between the normative solutions for the Rule of 72 and the structural coefficients for simple interest. Further investigation revealed that these two effects were due to differential responses to the training across the samples. To further characterize the differences in response to training, we examined subjects’ open-ended

responses, which revealed that some subjects in the control condition knew the Rule of 72 prior to coming to the experiment. The practice problems in the control condition primed this knowledge (which may not have been accessible prior to the seeing these problems), because the practice problems in the control condition were identical to the operations required for the Rule of 72 – they simply omitted the explanation of the steps. In addition, ordinary people found it more difficult to learn and/or implement the training procedure, which resulted in a greater percentage of trained subjects who failed to learn the Rule of 72 among ordinary people than among students. Table 2 shows that nearly three times the number of subjects in the Internet sample’s control condition used the Rule of 72 than in the student sample,  $\chi^2(1) = 3.77, p = .05$ , and a similar proportion failed to learn in the training condition,  $\chi^2(1) = 3.00, p = .08$ .

-----  
Insert Table 2 about here  
-----

We hypothesize that exposure to the Rule of 72 is stochastic – that is, exposure is not well-predicted by formal education, age, occupation (analyzed in broad categories), or highest degree earned. Instead, we speculate that exposure is a function of whether one has a specific interest in finance and whether one has been taught the rule (or sought it out). Apparently, the overwhelming majority of undergraduate business students have not been exposed to the rule. More people in the real world have been exposed, and they remembered the rule when they encountered our practice problems. Conversely, naïve undergraduates learned the rule better than ordinary people.

The short training procedure substantially increased accuracy in the evaluation of a related, but not identical, financial problem. The differences in retirement wealth between

Allison and Betty were large: 33% or \$100,000 in additional wealth. For a small cost in time, trained subjects were able to recognize the importance of the temporal dimension over the present value, reducing the probability of picking the incorrect answer by around 60%.

### **General Discussion**

We have demonstrated that people naturally anchor on simple interest when they are presented with a problem involving compound interest, which causes them to make large errors when they estimate the results of compounding processes. These errors are particularly egregious when the timeframe is long or when the interest rate is high, which are representative of the most important financial decisions that people are likely to make. We also demonstrated that compounding is asymmetric, in that prospective predictions are easier than retrospective estimates, regardless of what strategy one uses.<sup>8</sup> We proposed that this asymmetry is due to the increased difficulty of the math involved, and this prediction was supported.

Across all studies, individual level coefficient estimates confirmed that subjects who did not use the Rule of 72 instead anchored on simple interest. This was true both in “natural” conditions and in the training condition. Subjects who did not employ the Rule of 72 had mean coefficient estimates that matched the structural parameter estimates resulting from reporting simple interest, which is powerful evidence that the default anchor is in fact simple interest.

Finally, we showed that it is possible to train people to accurately use the Rule of 72 in a very short amount of time, which resulted in a 50% decrease in error. This result was qualified by noting that current undergraduates learned the Rule of 72 better than a sample of ordinary people. We conclude that it is relatively easy to train undergraduate business majors to use the Rule of 72, and that the same training procedure worked for around 86% of a diverse (and

---

<sup>8</sup> We replicated the Experiment 2 temporal frame manipulation using the design from Experiment 3. Details are available from the authors.

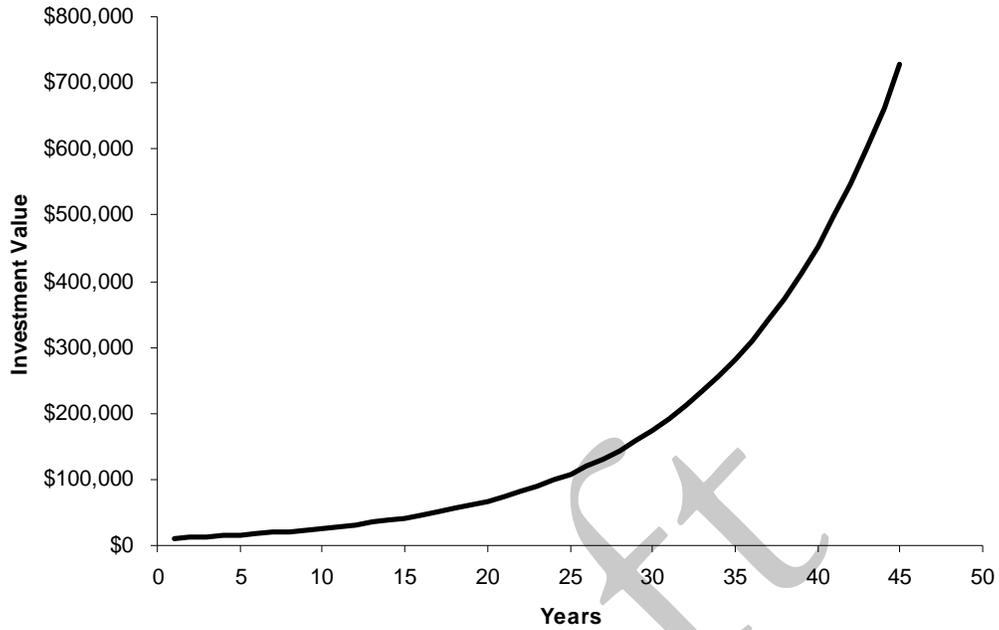
largely non-college educated) group as well. This learning translated into greater performance on a related problem involving compounding.

Future research should explore issue of the transfer of Rule of 72 training to other financial decisions involving compounding. Although we did observe transfer to a similar choice task involving retirement savings, it would be more interesting to investigate situations where the mapping is less apparent and/or where compounding is only one part of a larger, more complicated decision. It would also be interesting to compare training in the use of the Rule of 72 to the paternalistic interventions proposed by Benartzi and Thaler (2004) to encourage additional savings in both retirement and nonretirement investments. Finally, there is the obvious issue that in order to use the Rule of 72, people have to both remember the rule and remember to use it appropriately. Fortunately, the rule itself is not that complicated, so if people practice it hopefully they will be less likely to forget.

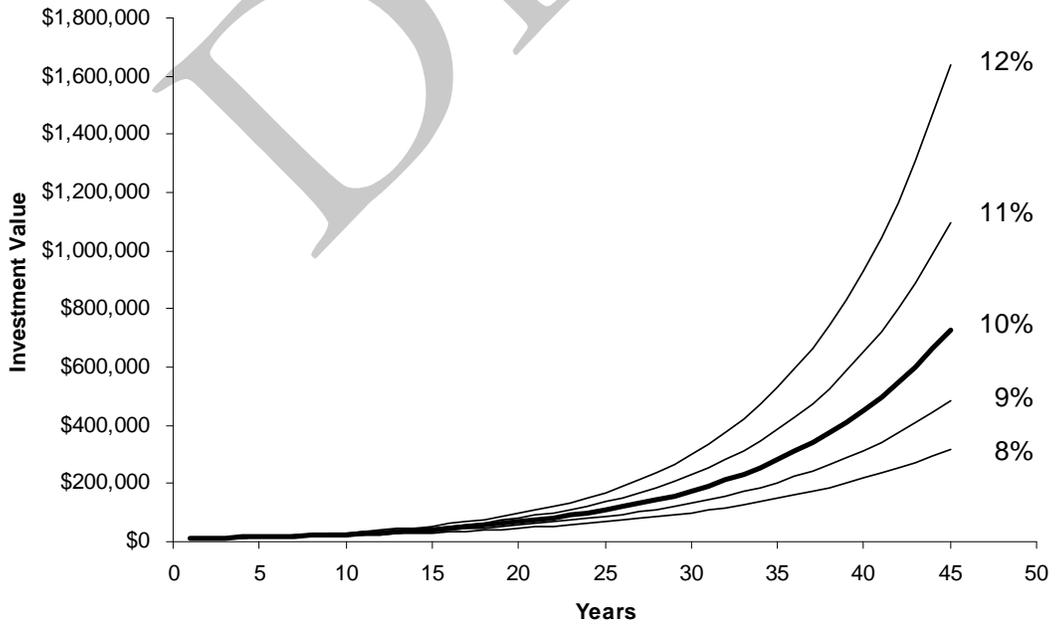
## References

- Benartzi, Shlomo and Richard Thaler (2004), "Save More Tomorrow: Using Behavioral Economics to Increase Employee Savings," *Journal of Political Economy*, 112 (1), 165-87.
- Birnbaum, Michael H. (2004), "Human research and data collection via the Internet," *Annual Review of Psychology*, 55, 803-32.
- Bolton, Lisa E., Luk Warlop, and Joseph W. Alba (2003), "Consumer perceptions of price (un)fairness," *Journal of Consumer Research*, 29 (March), 474-90.
- Eisenstein, Eric M. and J. Wesley Hutchinson (Forthcoming), "Action-Based Learning: Goals and Attention in the Acquisition of Market Knowledge," *Journal of Marketing Research*, 2006 (March).
- Fazio, Russell H. (1990), "A practical guide to the use of response latency in social psychological research," in *Research methods in personality and social psychology.*, Clyde Hendrick and Margaret S. Clark, Eds. Vol. 11. Thousand Oaks, CA: Sage Publications, Inc.
- Goldberger, Arthur S. (1968), "The Interpretation and Estimation of Cobb-Douglas Functions," *Econometrica*, 36, 464-72.
- Hoch, Stephen J. and Judy E. Tschirigi (1983), "Cue redundancy and extra logical inferences in a deductive reasoning task.," *Memory and Cognition*, 11, 200-09.
- Jones, Gregory V. (1979), "Generalized Polynomial Model for Perception of Exponential Series," *Perception & Psychophysics*, 25 (3), 232-34.
- Keren, Gideon (1983), "Cultural-Differences in the Misperception of Exponential-Growth," *Perception & Psychophysics*, 34 (3), 289-93.
- Nisbett, Richard E. and Timothy D. Wilson (1977), "Telling more than we can know: Verbal reports on mental processes," *Psychological Review*, 84 (3), 231-59.
- Wagenaar, William A. and Sabato D. Sagaria (1975), "Misperception of exponential growth," *Perception & Psychophysics*, 18 (6), 416-22.
- Wagenaar, William A. and Han Timmers (1978), "Extrapolation of Exponential Time-Series Is Not Enhanced by Having More Data Points," *Perception & Psychophysics*, 24 (2), 182-84.
- (1979), "The pond-and-duckweed problem; three experiments on the misperception of exponential growth," *Acta Psychologica*, 43, 239-51.
- West, Patricia M. (1996), "Predicting preferences: An examination of agent learning," *Journal of Consumer Research*, 23 (1), 68-80.

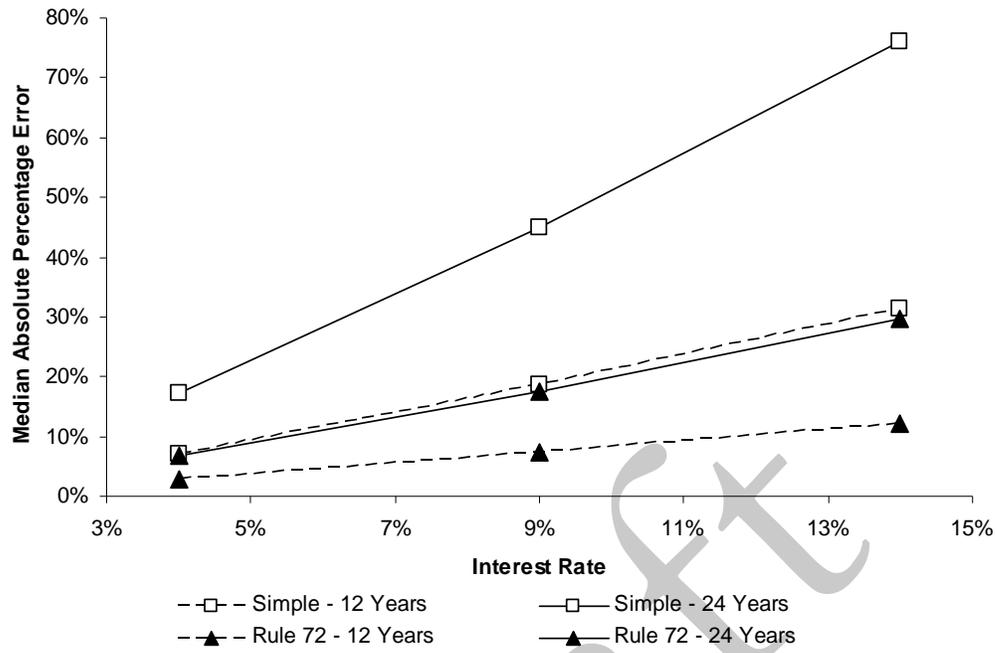
**Figure 1a: Compound Growth of a 10%/yr Investment**



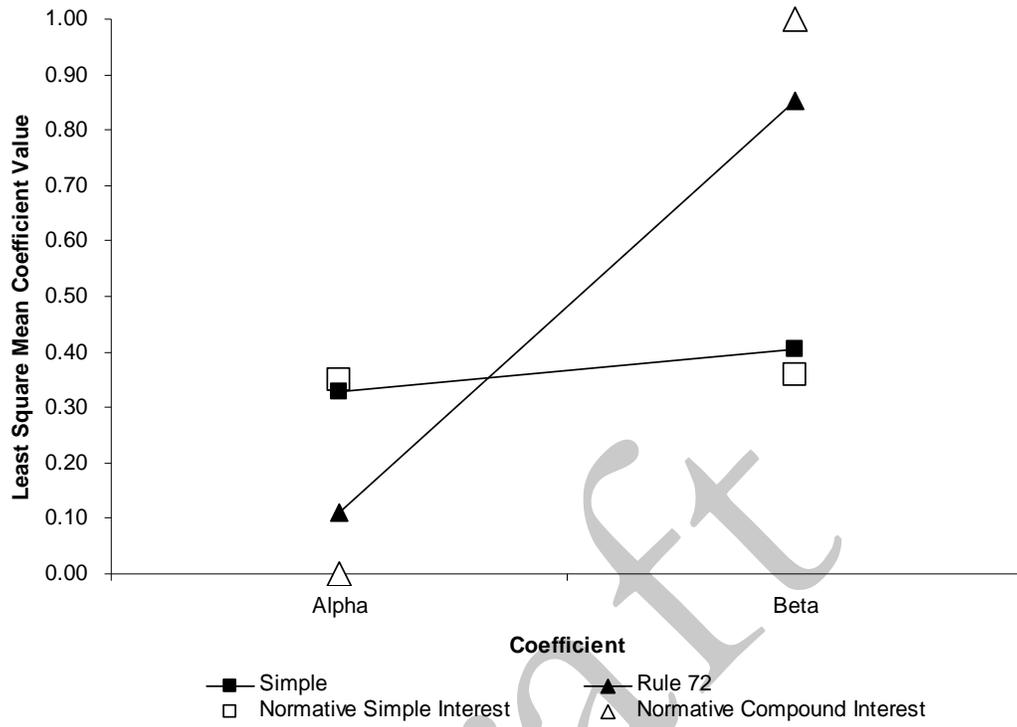
**Figure 1b: Total Returns for \$10,000 Invested at Different Rates of Return**



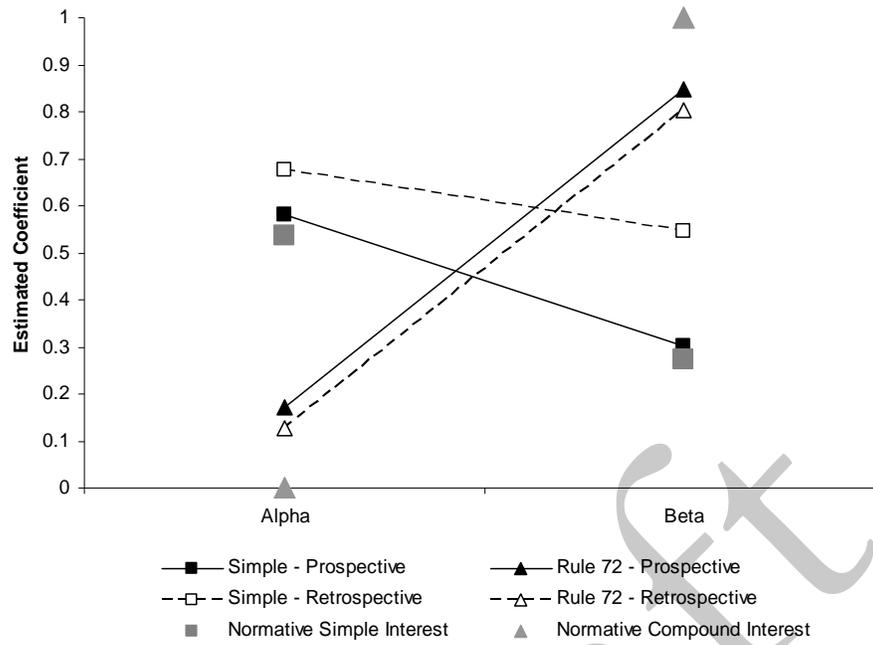
**Figure 2. Median Absolute Percentage Error of Prediction**



**Figure 3. Estimated coefficients for Rule of 72 vs. other strategies.**

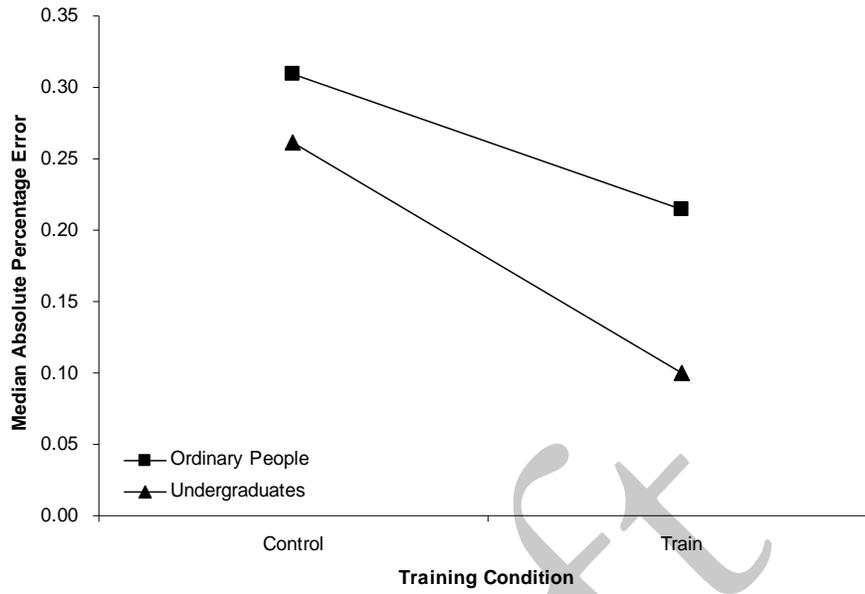


**Figure 4. Estimated coefficients for Rule of 72 vs. Simple Interest strategy**



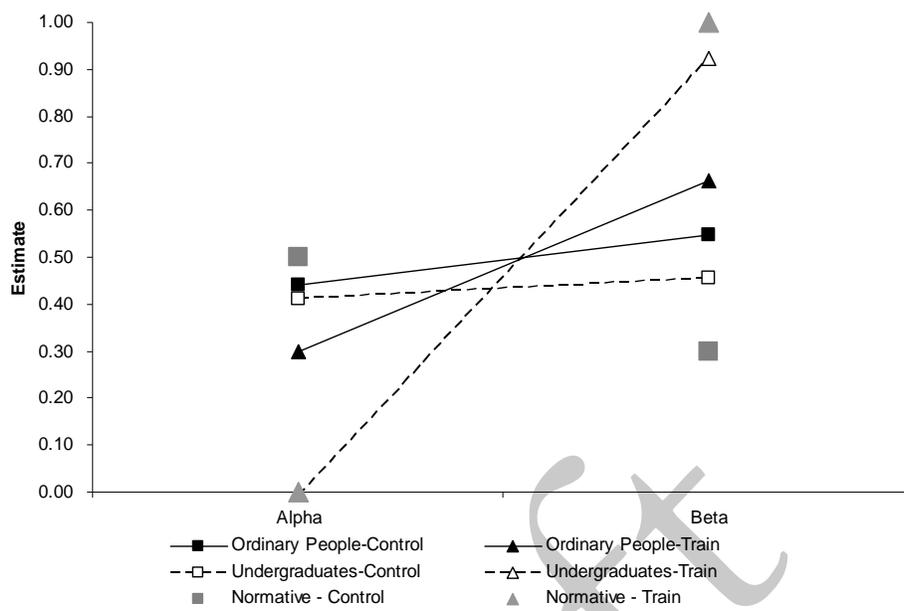
Draft

**Figure 5: Accuracy as a Function of Training and Sample**



Draft

**Figure 6. Mean Estimated and Structural Coefficients from Equation 3**



Draft

**Table 1: Estimated probability of picking the incorrect answer (Betty)**

| Sample          | Training Condition |       |
|-----------------|--------------------|-------|
|                 | Control            | Train |
| Ordinary People | 0.45               | 0.17  |
| Students        | 0.22               | 0.12  |

Draft

**Table 2: Comparison of strategy by experiment**

|   | <u>Undergraduate sample</u> | <u>Internet sample</u> |
|---|-----------------------------|------------------------|
| Percent in control condition using Rule of 72     | 4%                          | 12%                    |
| Percent in trained condition not using Rule of 72 | 4%                          | 11%                    |

Draft